## Numbers, Fixed Point Arithmetic and Complementary Numbers.

Introduction Maths and numbers are a subset of most spoken languages, however, not all spoken languages have a need to account for quantities and therefore the need for numbers is unnecessary. For example an individual may be asked 'How many Animals do you have ?'
In the society without numbers the replies may be 'None', 'One or possibly Two' or 'Many' whereas in the society with number they can give you definitive numeric response to your question.
As we can see Numbers and Maths and are only needed when a group of people need to perhaps trade, tax or identify their power, wealth or influence.

Complementary numbers system is fully compatible with standard calculation processes and is the basis of it with the only difference being that complementary numbers do not mirror around the value zero so negative numbers are presented differently. This allow you can perform additions, subtractions and sometimes multiplication's all within the same calculation process with ease as the processing rules do not need change.

Numbers are a representation way of identifying quantities. For example in the Roman times they used the following letters to represent values.
$\mathrm{I}=$ One $\quad \mathrm{V}=$ Five $\quad \mathrm{X}=$ Ten $\quad \mathrm{L}=$ Fifty $\quad \mathrm{C}=100 \quad \mathrm{D}=500 \quad \mathrm{M}=1000$
with intermediate value built up from combinations of the above letters.
To help keep the number letters used a small as possible in general when an smaller value letter prefixed a larger value letter it results in the lower value being taken away from the higher value letter.
For example $\mathrm{II}=$ Two. $\quad \mathrm{III}=$ Three. $\quad$ but $\quad \mathrm{IV}=$ Four $\quad \mathrm{V}=$ Five $\quad$ and $\mathrm{VI}=$ Six
The main disadvantage to the Roman numeral system is that it does not lend itself very well to mathematical processing and even quantity recording as there is no Zero to record absence of items.

Once the Arabic nations had devised a symbol for Zero the images below may explain the shape of the characters to their numeric value. (Hint: count the number of angles of each characters)


The Value Zero as it is circular has no angles so it is used to represent the numeric value zero.
So we now have our Ten character symbols and as we usually count in set of ten when recording quantities and other such activities can now be undertaken.

Note in maths and other disciplines the number of characters that you use for counting if often refereed to as the Base. So Base 10 means you use ten character symbols. In computers the maths uses Base 2 and only needs the character symbols Zero and One.
For completeness some computer systems also use Base 8 aka Octal as well as Base 16 aka Hexadecimal.

# Numbers ,Fixed Point Arithmetic and Complementary Numbers. 

## Counting

When we need to count objects we will generally collect them up into sets or piles of a defined amount that will assist us to calculate the total number of items. When we have a significant number of objects then we group the smaller sets into larger set to make totalling easier. The grouping continues until we are able to identify the exact total of all items being counted. A secondary reason for the grouping is to easily check if any counting errors have occurred. (A typical example is how coins are weighed in Banks to verify the bag contents match their expected contents..)

The structure of our numbers system follows a similar concept. Assuming we are using Base10 numbers then a digit in a column to the left represent values ten times greater than the same digit in reference column. The reverse applies when the comparison is to the right where it would be ten times less.
This is why when performing calculations we position the numbers in their correct columns.
This leads us on to devious maths techniques.
Assume we needed to multiply 53.27 by 19 .
As we know $19+1=20$ and $20=10+10$
As we know to multiply by 10 move all the digits in the number one place left or the decimal point one place right.
we could simplify our calculation to be :-
so 53.27 times $10=532.7$ (Just shift the digits left one place to multiply by 10) and with complementary number calculations the next operation is a single process.
$532.7+532.7-53.27=1012.13$

When we are counting/observing digits in columns we expect them to follow a sequence Example 1. counting up and observing just one column $0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5$ etc

Example 2. counting down and observing just one column $9,8,7,6,5,4,3,2,1,0,9,8,7,6,5,4$ etc

Example 3. counting down and observing just one column $9,8,7,6,5,4,3,2,1,0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5$ etc

With traditional taught maths all three examples are possible whereas only examples 1 and 2 are possible with complementary number calculations hence the simpler calculation rules.

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Fixed Point Arithmetic is based on the concept that all numbers used in your calculations must take the same shape. For calculation purposes three extra columns are also added to your number O,S and C. For example <Integer part is 10 characters wide><Fraction marker $\bullet$ < Fraction part is 4 characters wide>

$$
\text { OS:O:1:2:3 } 45: 6
$$

The width of the Integer and Fraction parts may be pre-set to any width that your calculations require. For all the following examples, I will assume we have decided to use $10 \bullet 4$ layout

Fixed Point Format value conversion (Note Cells O,S and C are for the moment left unused.)
If we have the value +2417895.306 then its Fixed Point Format would be :-
OS
To avoid column confusion alr empty digit column ceils would have Zero entered into them.

Complementary Numbers are an alternative way of identifying between positive and negative values. There is one exception to this concept in that the value Zero is neither positive nor negative but it will always be presumed to be positive. Note: No matter how many times you Add Zero to or Subtract Zero from a value, the value does not change. To complement a value, it is just a simple digit substitution exercise using the table below. (If Initial digit $=6$ then the Complementary digit is 3 )

| Initial Form | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Complementary Form | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

If we have the value -2417895.306 then its initial Fixed Point Positive Form would be :-

$$
O S O O O 24418795 \cdot 3060 \square C
$$

Now we can Complement the Digits and Change the C column to a 1 indicating that the number has been complemented. Finally the $S$ or Sign cell is changed to the same value as the first digit to its right.
Note: Even if we do not need to complement the number the $S$ or Sign cell is also changed to the same value as the first digit to its right and the C cell is set to a Zero. The C Cell is just a special carry in value.

$$
099997501204 \cdot 693911
$$

Summary of the negation process is (Complement/Invert the Digits then Add One).
The Calculation Process is the easy bit. Assume we wish to subtract 2417895.306 from 2417895.306 . As we already know both the positive and negative forms of the number, we can simply add the two known numbers together.

To complete the process, we now only need to do a bit of counting.
Initially, count all the values in the C column $(=1)$ and write that value in the Carry Lines Least Significant Digit Column. Next count the columns contents and any carries for all the remaining number columns.

## Calculation Set

+ Value


Carry Line
Result Line


$$
\text { This indicates the result }=0
$$

Note In the Result line the O cell is the overflow Indicator and can usually ignored the S cell indicates the values sign. When it equals 9 the number is Negative and 0 indicate Positive value. The overflow indicator is used to identify when a result number is too large to fit in the fixed length digit format.

## Numbers ,Fixed Point Arithmetic and Complementary Numbers.

## What happens when we complement the value zero.

Value $=0$
Complement


We now Count/Add all the ones in the C column and transfer that number to the Carry line.

Carry Line
Results Line


We now Count/Add the first Complement Column to the Carry Column.
So $9+1=10$ so 0 goes in the Results Column and 1 into the Carry Column
Carry Line
Results Line


Repeat the Count/Add Sequence for Next Column
Carry Line Results Line


Repeat the Count/Add Sequence for the rest of the Columns
and so on Results Line


As we normally can ignore the $O$ overflow column we can see that the complement of zero is zero.

## An example to try

+5324.8
-606.45
+205.22
-4923.57


Carry Line
Results Line


Although all the examples show O S C and Decimal point • columns with the calculations you do not need to use them once you are familiar with the process. All you need to be aware of is correct the position of the calculation digits as per traditional numbers.

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## Work Sheet



